March 1-3, 2010, California Institute of Technology, Pasadena, CA, USA

Analytical and numerical solutions for the onset of cavitation in rubber under unequal stresses

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1. Introduction

Physical evidence [1, 2] has shown that sufficiently large tensile loads can induce the sudden appearance of internal cavities in elastomeric solids. The occurrence of such instabilities, commonly referred to as cavitation, can be attributed to the growth of pre-existing defects into finite sizes. Because of its close connection with material failure initiation, the phenomenon of cavitation has received much attention from the materials [3] and mechanics [4] communities. Cavitation has also been a subject of interest in the mathematical [5] community because its modeling has prompted the development of techniques to deal with a broad class of non-convex variational problems. While in recent years considerable progress has been made via energy minimization methods to establish existence results [6, 7], fundamental problems regarding the *quantitative* prediction of the occurrence of cavitation in real elastomeric materials remain largely unresolved.

In this work, we concern ourselves with studying the effect of loading triaxiality on the onset of cavitation in rubber. Specifically, we derive analytical — by means of a new iterated homogenization method (see Section 2) — and numerical — by means of the finite element method (see Section 3) — results for the onset of cavitation in a Neo-Hookean solid when subjected to arbitrary 3D loading conditions. In this connection, it should be emphasized that the vast majority of cavitation studies to date have been almost exclusively limited to hydrostatic loading conditions, presumably because of the simpler tractability of this relevant but overly restricted case. However, the occurrence of cavitation is expected to depend very intricately on the triaxiality of the applied loading conditions, not just on the hydrostatic component [2, 8, 9, 10].

2. Iterated Homogenization Approach

In contrast to previous efforts mostly based on energy minimization methods, the idea here is to begin by casting the phenomenon of cavitation in rubber as the homogenization problem of nonlinear elastic materials weakened by random distributions of initially infinitesimal cavities (or defects) with initial porosity $f_0 = 0+$. Then, by means of a new iterated homogenization procedure [11, 12], we are able to construct *exact closed-form* solutions for such a problem. These include solutions for the change in size of the underlying cavities as a function of the applied loading conditions, from which we can readily determine the onset of cavitation corresponding to the event when the (initially infinitesimal) cavities suddenly grow into finite sizes (i.e., whenever the current porosity f in the deformed configuration is such that $f \gg f_0$). In spite of its generality, the relevant analysis of the proposed formulation reduces to the study of computationally tractable Hamilton-Jacobi equations in which, rather interestingly, the initial size of the cavities plays the role of "time" and the applied load plays the role of "space".

When specialized to the case of a standard incompressible Neo-Hookean rubber, with stored-energy function

$$W = \frac{\mu}{2}\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right), \tag{1}$$

weakened by a *random* and *isotropic* distribution of vacuous cavities, the iterated homogenization procedure leads to the following criterion: *inside a Neo-Hookean rubber*

The Japan Society of Mechanical Engineers, [No.10-204]

cavitation will occur at a material point P whenever along a given loading path the principal Cauchy stresses t_i $(i = 1,2,3)$ *first satisfy the following condition:*

$$S\left(t_1,t_2,t_3\right) = 8t_1t_2t_3 - 12\mu(t_1t_2 + t_1t_3 + t_2t_3) + 18\mu^2(t_1 + t_2 + t_3) - 35\mu^3 = 0\,, \quad (2)$$

where the constraints $t_i \geq 3\mu/2$ $(i = 1,2,3)$ must be enforced.

3. Finite Element Approach

Complementarily, in order to study the phenomenon of cavitation in solids with the Finite Element Method (FEM), we consider the problem of a single, initially spherical, vacuous cavity located at the center of a Neo-Hookean cube (see Fig. 1(a)) that is subjected to affine boundary conditions. After a parametric study, we found that in the context of this approach an initial porosity of $f_0 = 10^{-9}$ is sufficiently small to be representative of the limiting case of a spherical cavity in an infinite medium (corresponding to $f_0 = 0+$). Moreover, we found that cavitation can be identified to occur roughly whenever along any loading path of choice the current porosity f (i.e., the current size of the cavity) becomes 5 orders of magnitude larger than the initial porosity, namely, whenever $f/f_0 = 10^5$ (see Fig. 1(b)).

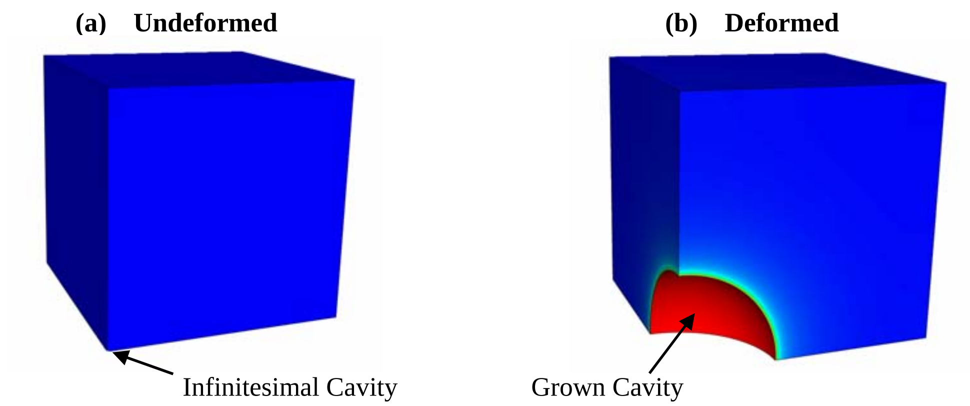
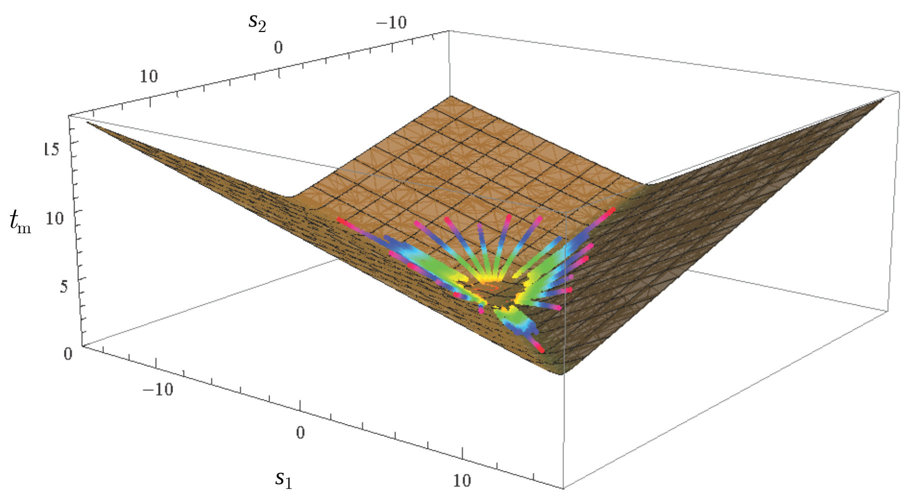


Fig. 1. FEM schematic illustrating the phenomenon of cavitation as the growth of an initially infinitesimal defect. In the undeformed state, the volume fraction (or initial porosity) of the defect is infinitesimal and remains infinitesimal along any given loading path up to a critical applied load at which it suddenly becomes of finite size. This event corresponds to the onset of cavitation.

4. Onset-of-Cavitation Surfaces



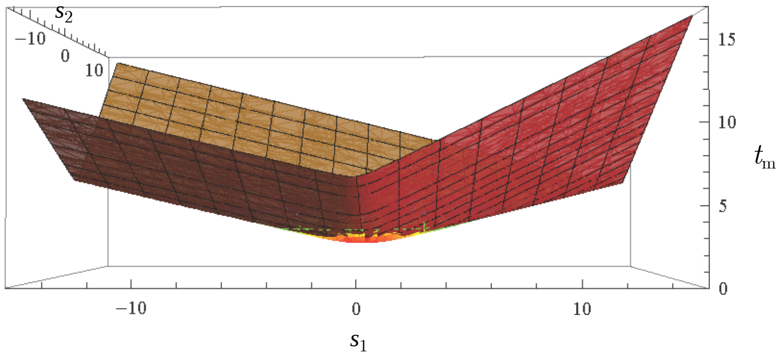


Fig. 2. Two views of IH (solid surface) and FEM (color dotted lines) onset-of-cavitation surfaces in Cauchy stress-space for an incompressible Neo-Hookean rubber. The vertical axes corresponds to the (normalized) hydrostatic stress $t_m = (t_1+t_2+t_3)/3\mu$, while the two horizontal axis correspond to the (normalized) deviatoric stresses $s_1 = (t_1-t_2)/\mu$ and $s_2 = (t_1-t_3)/\mu$.

Figure 2 presents two views of IH and FEM results for the critical hydrostatic $t_m = (t_1+t_2+t_3)/3\mu$ and deviatoric $s_1 = (t_1-t_2)/\mu$, $s_2 = (t_1-t_3)/\mu$ stresses at which cavitation ensues in a Neo-Hookean rubber. A main observation from these plots is that the onset of cavitation depends very critically not only on the value of hydrostatic stress but also on the value of deviatoric stresses. In particular, the larger the deviatoric stresses the larger the hydrostatic stress at which cavitation occurs. Another important observation from Fig. 2 is that the IH and FEM results are in very good agreement, in spite of the fact that they correspond to exact solutions for two very different initial distributions of defects. The derivation of these results and their physical significance will be further detailed during the oral presentation.

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